HIGH RESOLUTION ANALYSIS OF SATELLITE GRADIOMETRY

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ABSTRACT

Satellite gravity gradiometry is a technique now under development which, by the middle of the next decade, may be used for the high resolution charting from space of the gravity field of the earth and, afterwards, of other planets. This paper reviews some data analysis schemes for getting detailed gravity maps from gradiometry on both a global and a local basis. It also presents estimates of the likely accuracies of such maps, in terms of normalized spherical harmonics expansions, both using gradiometry alone and in combination with data from a Global Positioning System (GPS) receiver carried on the same spacecraft. It compares these accuracies with those of current and future maps obtained from other data (conventional tracking, satellite-satellite tracking, etc.), and also with the spectra of various signals of geophysical interest.

1. INTRODUCTION

A gravity gradiometer placed in a low, polar, near-circular orbit, will provide within a few months a fine global sampling of the gravity field. From such data, high resolution maps of the anomalous field (through 100 km half-wavelength) are expected, if the accuracy of each component of the gravity tensor measured by the instrument is of the order of 0.01 Eötvos or better (this means detecting a change of 10^{-12} m/sec^2 in gravity acceleration over a 10 cm distance). Both the level of resolution and the homogeneous quality of the results worldwide are beyond what can be obtained by any other space technique, except altimetry. It will have the advantage over altimetry of not being restricted to the oceans and of being more accurate at mid- and long-wavelengths, although fine resolution is likely to be considerably poorer. These conclusions follow from a number of studies (Balmino, 1985, NASA Gravity Workshop, 1987, Colombo, 1987, ESA Report, 1988, etc.). Those studies suggest that the gravitational effects of major oceanographic and geophysical features (such as global ocean circulation, mantle convection, spreading and subduction zones, etc.) could be charted with uncertainties of the order of 10% of their values at the earth's surface. The global character of the data offers the possibility of studying large areas now poorly known (Antarctica, the Artic, parts of South America and Africa), and also regions from which reliable information is not readily available (much of Asia). Employing gradiometers to chart the gravitational fields of other planets and their moons is a further step to take, once the major ones of building and using an earth-mapping instrument have succeeded.

2. DATA ANALYSIS

A successful gradiometer mission should collect millions of measurements over several months. To represent gravity worldwide to the expected resolution of better than 100 km, in the order of 100000 parameters have to be estimated, regardless of the type of base functions used (as long as they are generic functions, such as spherical harmonics, point masses, mean anomalies, and so on). There are two main categories of maps to be considered: global and local. A global map represents the field over the whole earth; a local map, the field over a limited region. Both kinds are complementary: a global map can be used for studies of the planet as a whole; its resolution may be less than the maximum allowed by the data, because the largest number of parameters that can be estimated in practice may not be enough to represent the smallest details everywhere. A local map may reach that maximum resolution over a limited area, because only enough parameters for this geographically restricted representation are needed. A good global map may be used to eliminate trends from the data employed in local maps, to help distinguish small details. Efficient schemes for analysing data, whether global or local, can exploit the uniform geographical distribution of the measurements resulting from the gradiometer's sampling of the field at regular intervals along an orbit with a repeating groundtrack. Such an orbit, with a repeat period comparable to the duration of the mission, provides the finest, most even coverage possible over that period.

2.1 Global Maps

The two most laborious numerical operations involved in estimating the parameters of a global gravity map by a least squares procedure are the formation and the inversion of the normal matrix of the estimator. With a repeating orbit, a regular, uninterrupted sampling of the field by a gradiometer, and the use as base functions of uniformly spaced point masses, or gravity anomalies, or else of spherical harmonics, the elements of the normal matrix can be calculated directly by analytical expressions (instead of being accumulated observation by observation). The matrix also has a very strong structure that allows its fast inversion and limits roundoff error propagation, in spite of its enormous size (order of 10^{10} elements). This structure may be block-diagonal, and very sparse (with spherical harmonics), or block-Toeplitz circulant, and highly redundant, (with gridded point masses and mean anomalies). In the case of spherical harmonics, the block-diagonal structures occur when the potential coefficients are separated first by order, then by parity (sine or cosine). For a perfectly circular orbit, a third symmetry exists: that of harmonics where n-m is even versus those where n-m is odd (n is the harmonic degree and m the order). This type of structure, as well as the block-Toeplitz one, is not exclusive to gradiometry, but arises whenever these base functions are used together with equispaced gravity data sampled along a repeating orbit. They are discussed in Colombo (1981), (1984), and (1987), and in Wagner (1983). In the case of spherical harmonics, the representation of the potential along a repeating orbit is a Fourier series with the same repeat period as the orbit; in this series, harmonics of different orders, parities, etc., have no common frequencies. This causes many columns of the matrix of observation equations associated with the potential coefficients to become orthogonal when the sampling rate is much higher than the frequency content in the signal. It is also necessary to take care of time-varying components of the signal that do not share the periodicity of the orbit. As explained in Colombo (1984), most of the orbital perturbations due to the anomalous gravity field repeat with the orbit (so they are "geographically correlated": they are the same every time the spacecraft passes over the same place, in each successive repeat). There are, however, deep resonant perturbations (caused mostly by the zonals) that will vary from one repeat to the next; there are also rotational effects, instrumental drift, etc., that lack this periodicity. They can be accomodated by means of additional parameters associated with suitable non-periodical time functions, to be estimated together with the potential coefficients. The result, for spherical harmonics, is an "arrow" normal matrix, as shown in Figure 1. The two edges of the "arrowhead" are non-zero elements associated with both potential coefficients and with the additional parameters; the "shaft" of the arrow is a string of diagonal blocks of decreasing size associated with the potential coefficients alone. This matrix can be inverted efficiently and accurately by the method explained in Colombo (1984).

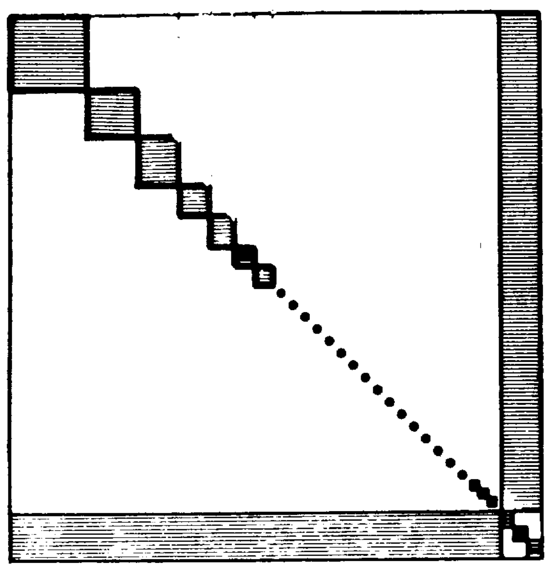

Figure 1. Sparse "arrow" structure of the normal matrix of spherical harmonic potential coefficients for a repeating orbit. White areas show location of zero elements.

2.2 Local Maps

If the data has been sampled without significant interruptions (a few small gaps can be filled by simple interpolation, to restore symmetry, without significant degradation of results), a variety of efficient data analysis techniques can be used; a few examples of these are given here. By separating ascending from descending passes, and chosing a region with parallels of latitude as the northern and southern limits, and groundtracks of passes as the eastern and western edges, equispaced points along the remaining passes define a twisted regular grid on this region. Using this grid to place the point masses or mean anomalies of a local representation leads to an adjustment of their values where the normal matrix is block-Toeplitz. Moreover, in equatorial regions, for sufficiently small areas (where the tracks are virtually straight and parallel, and the earth can be treated as flat), two-dimensional Fourier expansions can be used to represent the field, and fast Fourier transform procedures implemented to estimate the parameters of the local maps.

3. MISSION ANALYSES

Mission analyses based on a global data coverage using instruments measuring several components of the gravity tensor over a six-month period, with sampling/averaging data rates of a few seconds, indicate that the sensitivity of the gradiometer initially increases with wavelength, as one would expect from the frequency response of a twice-differenced process (the gravity potential), and then decreases as the spectrum of the potential decays nearly exponentially with frequency, primarily due to the effect of altitude. The spherical harmonic representation of the field has been used in all cases, assuming equispaced measurements along a circular, polar, repeating orbit, to use the efficient methods outlined in section 2.1 for setting up and inverting the normal matrix to obtain the variance-covariance matrix of the estimated potential coefficients.

Figure 2 shows the accuracy of the coefficients estimated with data from a 0.01 Eötvos (E) instrument with a 4 second sampling/averaging rate. A number of spurious effects, mostly of low frequency, such as: orbit errors, instrumental drift, effects of rotation, attitude error, etc., have been suppossedly removed by filtering out all information below three cycles per orbital revolution (i.e., only signal with periods shorter than 30 minutes is left). This leads to conservative error estimates. For the gradiometer, two curves, one for an accuracy of 0.01 E, and another of 0.0001 E (the expected accuracy of the supercooled instrument under development for NASA by Ho Paik at the University of Maryland) are shown. Both indicate an initial steep increase in sensitivity, followed by a botttoming-out at about degree 90, and experiencing a nearly exponential increase (approximately straight line in the linear-log graphs shown here) beyond degree 200. The divided curve marked "0.01 E+GPS, best and worst cases" corresponds to the combination of the gradiometer measurements with GPS tracking data in the estimation of the gravity field model. This assumes that a GPS receiver is carried on board of the gradiometer satellite, and that it tracks continuously the GPS spacecraft from the full constellation, simultaneously with a ground GPS network of some 10 globally distributed stations (in order to correct the GPS clocks and orbits, as well as the satellite receiver clock, by double differencing of observations (Yunk et al., 1985)). This not only strengthens the gravity field solution at low degrees (here, through degree 30), but also helps the operation of the ground network (primarily set up for geophysical experiments on geodynamics, according to current expectations for the 1990's) by reinforcing the geodetic connections between the widely separated stations. In this way, two different geodetic and scientific tasks can be combined to mutual advantage. The "worst case", upper curve, assumes large orbit errors, and considerable unresolved carrier-phase biases (order of 10 cm). The "best case", lower curve, corresponds to the ideal situation of no orbit errors either for GPS or for the gradiometer satellite, and no unresolved biases. (For further details on the GPS calculations, see extended abstract on GPS/GP-B by Smith et al., in this issue.) In general, the assumptions were: both the L1 and L2 frequencies are used, with carrier-phase and pseudorange measured; up to seven GPS satellites simultaneously visible from the lower satellite at all times; average GDOP of 3; data corrected for ionospheric refraction and then double differenced; assumed measurement noise: 1 cm for carrier-phase, and 10 cm for pseudorange (after fifteen minutes averaging) in the worst case; 1 cm for carrier-phase, and zero cm for pseudorange in the best case.

Clearly, GPS data and gradiometry complement each other, as the former is more sensitive to lower frequencies, and the latter to higher ones. The 0.0001 E curve is the same as for 0.01 E, but shifted down two decades. At the top of Fig. 2, the broken curve represents the power spectrum of the anomalous field, or full spectrum of the signal. Where the plot of the accuracies of the spherical harmonics intersects this spectrum, signal and noise are equal, and this is usually adopted as the point of highest recoverable frequency. For the 0.01 instrument, this point happens at about degree 360, for a half wavelength of some 70 km.

Figure 3 shows the curves for gradiometry/GPS of Figure 2 against the published accuracies of current, low resolution models such as GEM-T1 (top left corner), and those expected for the improved models planned for the early 1990's, which will include satellite altimetry as well as conventional satellite tracking data. Improvements over such models from the combination of GPS data and gradiometry are shown to be about two orders of magnitude in accuracy, and nearly one order of magnitude in resolution (fields like GEM-T1 and its planned successors are limited to resolutions of some 400 km half wavelength). Also plotted are the accuracies expected from the superconducting gradiometer (0.0001 E) and from satellite-satellite radio-tracking systems like the one designed for the now shelved GRM mission. Figure 3 suggests that a 0.01 E instrument will be better than GRM for features with a harmonic content above degree 120 (i.e., 180 km half wavelength).

The curve at the top of Figure 4 is the spectrum of the stationary sea surface topography (based on the work of Levitus on global circulation). Also shown is the spectrum of the M2 ocean tide based on the model of Schwiderski, and that of changes in the field over the length of the mission due to post-glacial crustal rebound and sea level adjustment. The plots indicate that major ocean tides can be estimated accurately (in fact, due to orbital resonances not considered in the calculations, tides can be estimated better than suggested here). Post-glacial rebound is barely observable over six months with a 0.01 E instrument, and a 0.0001 E device may be required for its study. The geoid calculated from gradiometry should be good enough to separate most of the stationary ocean topography from the mean sea surface determined with satellite altimetry.

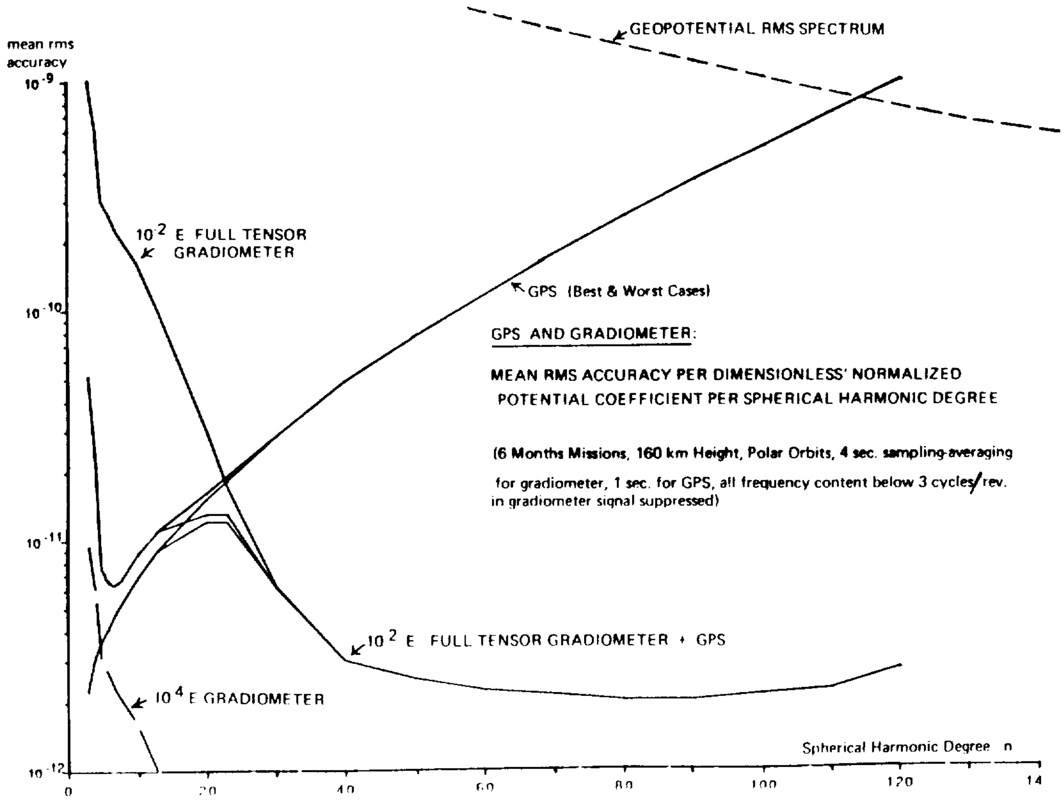

Figure 2. Accuracy per estimated potential coefficient per spherical harmonic degree (the spherical power spectrum of the coefficient errors).

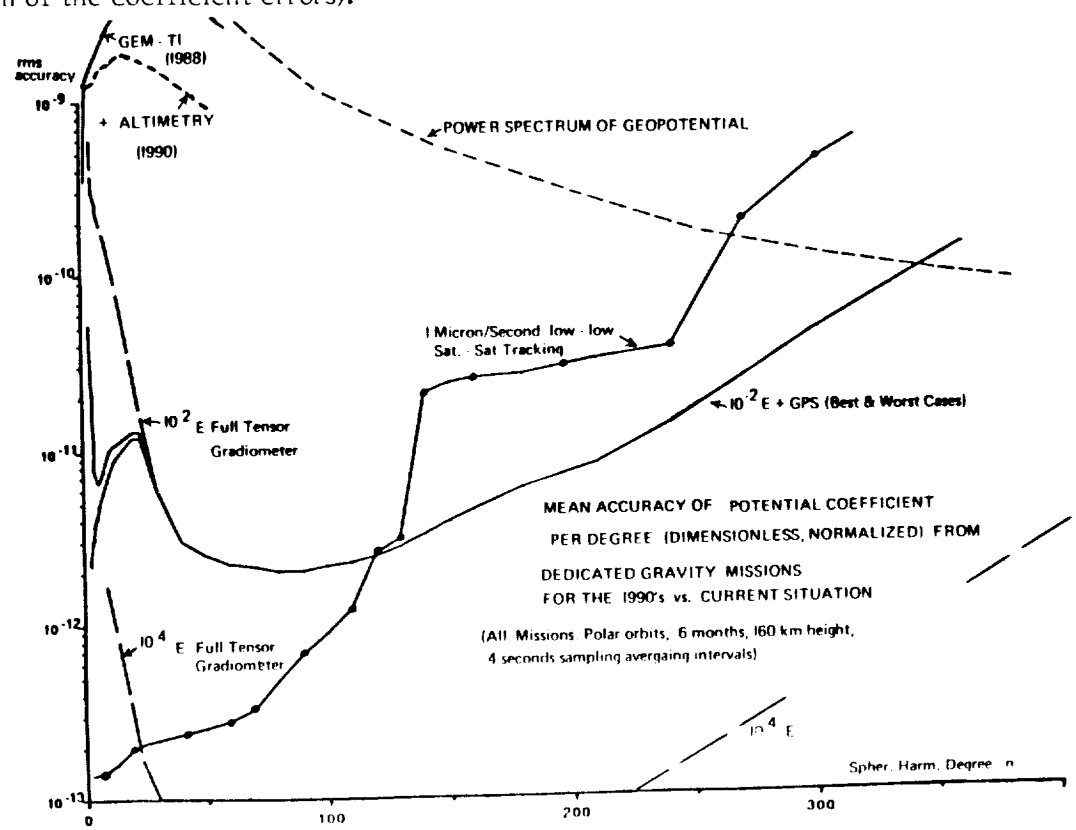

Figure 3. Accuracies of potential coefficients estimated from gradiometry and GPS, compared to present and future results derived from other data.

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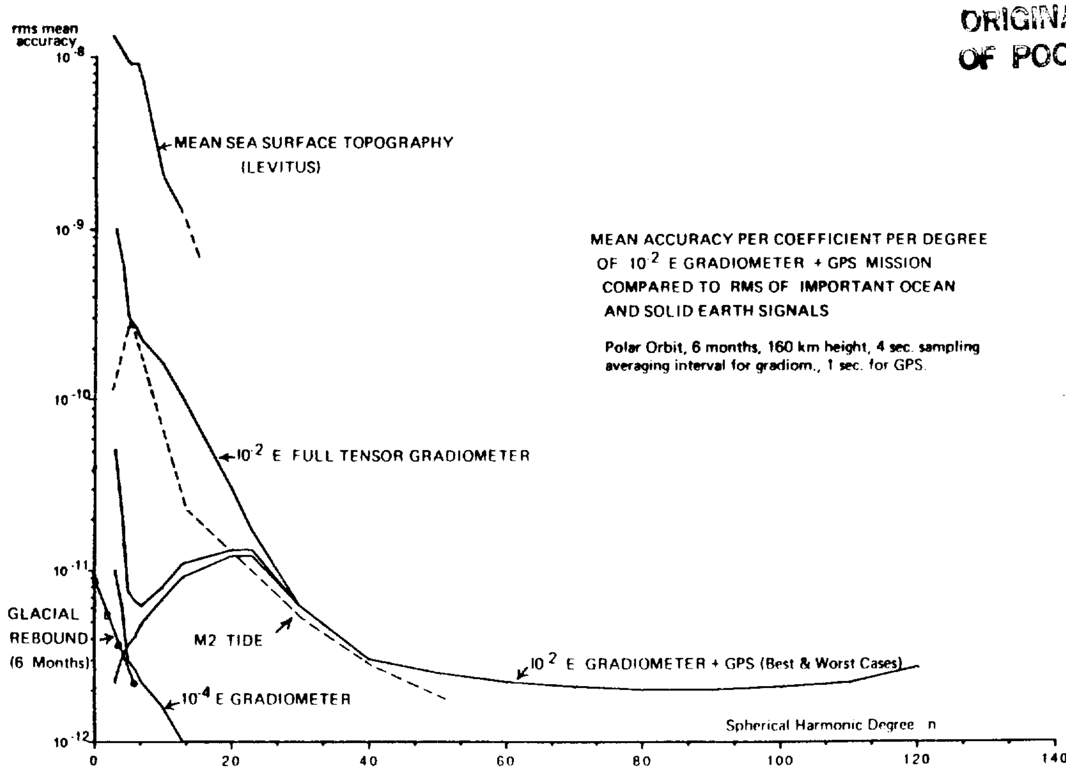

Figure 4. Comparing the accuracies of potential coefficients recovered from gradiometry, alone and in combination with GPS data, to the spectra of various phenomena of interest in oceanography and solid earth geophysics.

4. CHARTING THE GRAVITY FIELDS OF OTHER MEMBERS OF THE SOLAR SYSTEM

Gravity and topography can be mapped from space by means of gradiometry and altimetry; jointly, they can help identify the geological structures underlying the surface of a planet. Devices such as radar altimeters and gravity gradiometers may be carried in future space probes destined to orbit other members of the solar system, including the moons of the major planets. Because very little is known about those celestial bodies, instruments many times less sensitive than those needed to gain new information on the earth, may provide nevertheless a wealth of new knowledge. Since many planets and moons have little or no atmosphere, expensive, heavy, complex drag-compensating systems are largely unnecessary, and spacecraft can stay in lower orbits for longer periods than in Earth. Many planets are smaller than our own, but their geophysical features are of the same size or even larger than those on Earth. All these factors tend to compensate for any loss in instrument sensitivity caused by the more stringent limits on weight, size, volume, power consumption, durability, and cost, as well as from a less benign enviroment (e.g., equipment with moving parts in close proximity, etc.), likely to characterize interplanetary missions.

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REFERENCES

Colombo, O.L., The Ohio State University, Dept. Geod. Sc. and Surv., Report 310, Columbus, Ohio, 1984.

Colombo, O.L., Netherlands Geodetic Comission, Publications in Geodesy, New Series, Vol. 7, No. 3, Delft, Holland, 1984.

Colombo, O.L., Proceedings IUGG XIX General Assembly, Vancouver, August 1987.

Balmino, G., et al., SESAME report, Executive Summary for ESA, September 1985.

NASA Gravity Worshop, Colorado Springs, Colorado, February 1987.

ESA Executive Report on Aristoteles: A european solid earth mission, Noordwijk, Holland, July 1988.

Wagner, C. A., J.Geophys. Res., Vol. 88, pp. 10309-10322, 1983.

Yunk, T.P, Melbourne, W., C.L. Thorton, IEEE Transactions on Geoscience and Remote Sensing, Vol. GE-23, 4, pp. 450-457, July 1985.